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Risk spillover between financial markets based on non-parametric quantile Vine-Copula model

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ABSTRACT

Brics (China, Brazil, Russia, India, South Africa) as the world's emerging economies, the stability of their financial markets has an important impact on the global economy. This paper intends to study the correlation structure and risk contagion effect among the financial markets of the BRICS countries. Based on the stock market, an important indicator of the financial markets of each country, the paper uses the non-parametric quantile model to fit the marginal distribution of the returns of each country, constructs the D-Vine Copula model, explores the correlation structure among the financial markets of each country, and uses CoES to describe the risk spillover intensity. The empirical results show that China's financial market has significant risk spillover effect on other BRICS countries, and the impact on Russia is the most prominent, followed by India and South Africa, and the impact on Brazil is relatively weak and presents obvious asymmetry. These results have important theoretical value and practical significance for monitoring the risk contagion effect in BRICS countries.

KEYWORDS

Brics; Vine-Copula; Nonparametric quantile regression; Risk spillover

1 Introduction

BRICS countries (China, Brazil, Russia, India, South Africa) as the world's emerging economies, the stability and development of their financial markets play a key role in promoting global economic growth, not only related to their own economic prosperity and social stability, but also a profound impact on the direction of the global economy. The in-depth study of the relevant structure and risk contagion effect between them is helpful to understand better the operation mechanism of the global financial system and the law of risk transmission.

Cheng et al. (2006)[1] analyzed the development potential and challenges faced by the BRIC economies. It reveals the advantages and disadvantages of each. Chen Yangyang (2014)[2] studied the net contagion mechanism of extreme risks in financial markets among BRICS countries and analyzed the net contagion mechanism of extreme risks spillover and the components of net contagion. Luo Jia (2017)[3] discussed the four waves of global exchange consolidation since the 1990s and their impact on the BRICS countries and listed many examples to illustrate the difficulties faced by the BRICS countries in integrating financial resources.

With the continuous development and innovation of the global financial market, new financial products and financial instruments continue to emerge, and the complexity and uncertainty of the financial market are also increasing. Copula function has been introduced into the study of financial risk contagion effect with its unique advantages. Sklar(1959)[4] proposed Sklar's theorem, which laid a theoretical foundation for the wide application of Copula functions. Later, many scholars applied it to the financial market. For example, Yang et al. (2013)[5] used the GARCH-copula model to study the dependency structure among international stock markets. However, with the increasing complexity of financial markets, it is difficult for a single Copula function to describe the nonlinear dependence relationship between high-dimensional variables (Fan et al. (2001)[6]). Bedford et al. (2002)[7] proposed the Vine-Copula model, which constructs a series of pinion-connected binary Copula functions. Later, Brechmann et al. (2012)[8] used the Vine-Copula model to study the nonlinear dependence between financial assets and revealed the complex network characteristics of risk spillover. Xu Jiaqing et al. (2022)[9] studied the relevant structure of financial markets in BRICS countries against the background of COVID-19. Vine-Copula model, with its high flexibility and explainability, describes the multi-level and asymmetrical dependence and dynamic changes among variables, but it cannot show the nonlinear relationship between financial markets from multiple perspectives. Therefore, some scholars use quantile regression to capture the nonlinear relationship between financial markets under different conditions. Kraus D et al. (2017)[10] used the semi-parametric quantile D-Vine Copula to analyze the relationship between international banks and insurance CDS spreads. Most studies combining quantile and Copula are based on parametric or semi-parametric methods. Ye Wuyi et al. (2018)[11] found that the flexibility of the parameterized Copula function was limited. Non-parametric quantile methods are not limited by specific distribution assumptions or parameters and can more accurately reflect the true distribution of variables, including the impact of extreme events.

In addition, to accurately evaluate the risk spillover effect between financial markets, the measurement methods have been developed from simple linear methods such as correlation coefficient. Adrian and Brunnermeier(2016) [12] used a quantile regression model to calculate CoVaR to assess the risk spillover effect among different financial institutions in the United States. Girardi and Ergun(2013)[13] proposed generalized CoVaR and measured it with the GARCH model to explore the impact of American financial institutions on systemic risk. However, given the lack of monotonicity and subadditivity of CoVaR, some scholars have proposed CoES methods to overcome the limitations of CoVaR. Zhang Bingjie et al. (2018) [14] measured CoES based on a quantile regression model. Cao Jie (2021)[15] used the GARCH model and Copula function, respectively, to measure generalized CoES. So, this paper adopts the CoES method to measure the risk spillover effect.

To sum up, the study of the relevant structure and risk contagion effect among the financial markets of BRICS countries not only has important theoretical value and practical significance but is also of great significance for us to better understand and cope with global financial risks and promote economic development. The rest of the paper is organised as follows: In Section 2, the sigma-pi-sigma neural network and the batch gradient algorithm based on graph regularization are described in detail. Section 3 analyses the experimental results. Finally, Section 4 gives some conclusions of this paper.

2 The proposed approach

2.1 Fitting of edge distribution

In this paper, a nonparametric quantile model is used to fit the edge distribution, and the continuous kernel density estimation proposed by Parzen et al. (1962) [16] is adopted. It provides a smooth density estimate for each observed sample$X_i\,(i=1,\cdots,n)$, constructs a local estimate by weighted averaging adjacent data points, and the weight is determined by a kernel function that decays with distance so that not only the local features of the data are preserved, but discrete jumps are avoided. The accuracy of the analysis can be improved when dealing with complex data structures. The nonparametric functions in the set of functions representing the covariates are represented as follows:

$$f(x)=\frac{1}{n}\sum_{i=1}^{n}K\left(\frac{x-x^{(i)}}{h}\right),x\in R \tag{1}$$

Where $K(x)=\int_{-\infty}^{x}k(t)d(t)$is the symmetric probability density function, and $h>0$is the bandwidth parameter. Thus, the marginal distribution function estimation of $\widehat{F_x}$ and $\widehat{F_Y}$ obtained.

2.2 Vine-Copula model

Bedford and Cooke(2001) [17] proposed Vine Copula, which deals with multivariate dependencies through tree-like structure. The structure of D-Vine is characterized by linear distribution, and there is no obvious dominant factor between nodes. Given the market relationship of BRICS countries and subsequent empirical results, D-Vine can more intuitively explore the complex correlation structure among countries, so the D-Vine Copula model is adopted in this paper. The function F_Y represents the edge distribution of the response variable$Y$, and the function$(F_1,F_2,\cdots,F_n)$ represents the joint distribution of the variable$(X_1,X_2,\cdots,X_n)$. The conditional density of D-Vine is defined as:

$$f_{0|1,\cdots,n}(y|x_1,\cdots,x_p)=\prod_{j=2}^{n}c_{0,j;1,\cdots,j-1}\left(F_{0|1,\cdots,j-1}(y|x_1,\cdots,x_{j-1}),F_{j|1,\cdots,j-1}(x_j|x_1,\cdots,x_{j-1})\right)$$
$$\cdot c_{0,1}(F_Y(y),F_1(x_1))\cdot f_Y(y). \tag{2}$$

Where $F_{0|1,\cdots,j-1}$and $F_{j|1,\cdots,j-1}$represent the distribution function of the sum of conditional random variable, and for ease of estimation, the conditional distribution pair-Copula is assumed to be independent of the variable values on which they depend.

2.3 Calculation of generalized multidimensional CoES based on the Vine-Copula model

Cao Jie proposed generalized multidimensional conditional tail expectations (CoES), aiming at a more comprehensive and flexible assessment of the comprehensive risk spillover effect of multiple markets on another market under different risk scenarios, which is defined as:

$$CoES_{\alpha,\beta,\gamma,t}^{i|j,k}=E[R_{i,t}|R_{i,t}\leqslant CoVaR_{\alpha,\beta,\gamma,t}^{i|j,k}|R_{j,t}\leqslant VaR_{\alpha,t}^{j},R_{k,t}\leqslant VaR_{\beta,t}^{k}]=\frac{1}{\gamma}\int_{0}^{\gamma}CoVaR_{\alpha,\beta,q,t}^{i|j,k}dq. \tag{3}$$

CoES focuses on tail risk assessment, and more importantly, generalized multidimensional CoES effectively solves the problem of large fluctuations in risk spillover results due to changes in confidence intervals due to the nature of its continuous function, thus ensuring the robustness of the results. The above model studies the impact of two financial markets on the single stock market, that is, the stock market and the multiple risk spillover intensity of the market. Based on this, this paper extends the model to study the impact of multiple financial markets on a single stock market:

$$CoES^{i|j,k,l,m}_{\alpha,\beta,\theta,\mu,\gamma,t} = E[R_{i,t}|R_{i,t} \leqslant CoVaR^{i|j,k,l,m}_{\alpha,\beta,\theta,\mu,\gamma,t}|R_{j,t} \leqslant VaR^{j}_{\alpha,t}, R_{k,t} \leqslant VaR^{k}_{\beta,t}, R_{l,t} \leqslant VaR^{l}_{\theta,t}, R_{m,t} \leqslant VaR^{m}_{\mu,t}] = \frac{1}{\gamma}\int_{0}^{\gamma} CoVaR^{i|j,k,l,m}_{\alpha,\beta,\theta,\mu,q,t} dq \tag{4}$$

Where R_i, R_j, R_k, R_l, R_m represents the return rate of the five markets. $VaR^{j}_{\alpha,t}$, $VaR^{k}_{\beta,t}$, $VaR^{l}_{\theta,t}$, $VaR^{m}_{\mu,t}$, is the value at risk (and maximum possible loss) of these four markets at the confidence level and the $CoES^{i|j,k,l,m}_{\alpha,\beta,\theta,\mu,\gamma,t}$ is the maximum possible loss for market i at a given confidence level when all four other markets are at extreme risk.

This model represents the average additional loss that market i and the other four markets j,k,l, and m may face when there is an extremely adverse situation, in other words, the additional loss that market i may face when the four related markets perform badly. In other words, the excess expected tail loss is used to measure the multidimensional risk spillover effect when it occurs. The edge distribution function is known, and CoES is calculated by inverse solution.

3 Smulation results

3.1 Data source

In order to study the relevant structure and risk spillover effect of financial markets in BRICS countries, the representative stock markets of financial markets of each country are selected as Shanghai Index (000001.SH) in China, MOEX index (IMOEX.MCX) in Russia, SENSEX Index in Mumbai (SENSEX.bo) in India, and Shanghai Shanghai Index (000001.SH) in Russia. South Africa's Johannesburg All Share Index (JALSH.GI), Brazil's Bovespa index (IBOVESPA.GI), the data set spans from April 20, 2020 to April 19, 2024, excluding non-trading days to construct a dataset containing 869 trading days. Taking into account the differences in trading hours of the respective BRICS stock markets, the time period in which each country's stock markets were open was selected to ensure the synchronization and comparability of the data set, which included the COVID-19 phase and provided a reference for assessing the performance of each country's markets under extreme external shocks. Data from Wind database.https://www.wind.com.cn/portal/zh/EDB/index.html.

3.2 Descriptive statistics

Define the daily yield:

$$x_{i,j} = \frac{p_{i,j}}{p_{i-1,j}} - 1, i = 2,\cdots;T, j = 1,\cdots;M, y_i = \frac{p_i}{p_{i-1}} - 1, i = 2,\cdots;T.$$

Where, $x_{i,j}$is the yield rate of the constituent stock, $y_i$is the yield rate of the standard index, $p_i$is the closing price of the standard index at the moment$i$, $p_{i,j}$is the closing price of the stock j at the moment$i$, T represents the trading day, represents the number of markets, in this paper M=5.

3.3 The fitting of margin distribution of return series

Non-parametric quantile regression is used to fit the return distribution of the BRICS stock markets, and the results are shown in Figure 1.

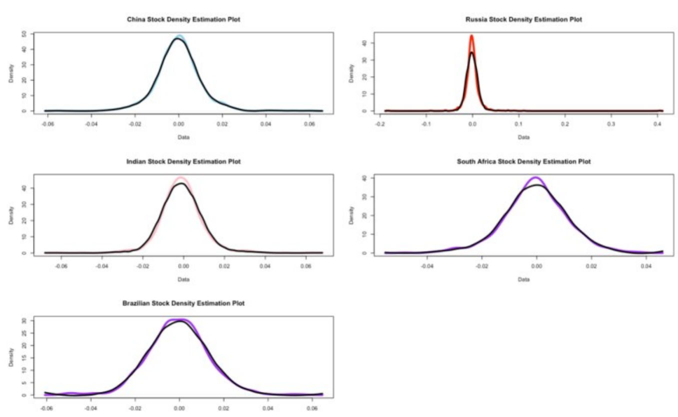

Figure 1 Fitting of the edge distribution of the non-parametric quantile regression model

The optimal bandwidth identified by the R language is 0.2, 0.1, 0.3, 0.4, 0.4. It can be seen that although the distribution shape of stock return in each country is different, the model in this paper can capture the nonlinear characteristics of the data well and show a good fitting effect. In addition, these graphs also show the distribution of stock returns in various countries, which helps to understand the risk and return characteristics of stock markets in various countries.

3.4 Establishment of the D-Vine Copula model

The D-Vine Copula model is used to describe the relevant structure of the five major markets. Four progressive tree layers are modeled, each representing a different order of conditional dependence structure, to represent the complex interactions between national markets. At the same time, the binary Copula in the first tree is optimized according to the BIC criterion. The binary Copula of the latter tree is optimized step by step according to the optimization results of the previous tree. The order of nodes is determined by the sum of correlations between nodes. The node with the largest sum of rank correlation coefficients with other nodes is regarded as the root node and listed in the first place and then numbered in descending order according to the rank correlation coefficients between other nodes and this root node (REN et al. (2015) [18]). Multiple Copula functions such as Frank, t-Copula, Normal, and Survival Gumbel are available to connect nodes to accommodate different correlation and tail-dependent properties. The Copula function that maximizes the likelihood function (logic) or minimizes the information criteria (AIC and BIC) is selected. For the sake of description, let 1 to 5 represent five stocks: 1 is China's Shanghai Index (000001.SH), 2 is Russia's MOEX index (IMOEX.MCX), 3 is India's Mumbai SENSEX Index (Sensex.bo), 4 is South Africa's Johannesburg All Share Index (JALSH.GI), and 5 is Brazil's Bovespa index (IBOVESPA) .gi), the results are shown and Table 1.

Table 1 The result of D-Vine Copula model

tree	edge	family	Copula	par	par2	tau	utd	ltd
1	3,1 |	5	F	0.65	-|	0.07	-	-
	2,3 |	2	t	0.21 (0.04)	9.28 |	0.13	0.03	0.03
	4,2 |	14	SG	1.15	-|	0.13	-	0.17
	5,4 |	14	SG	1.1	-|	0.09	-	0.12
2	2,1|3	5	F	0.47	-|	0.05	-	-
	4,3|2	1	N	0.14	-|	0.09	-	-
	5,2|4	1	N	0.09	-|	0.06	-	-
3	4,1|2,3	1	N	0.06	-|	0.04	-	-
	5,3|4,2	3	C	0.07	-|	0.04	-	-
4	5,1|4,2,3	2	t	0.05	20.39 |	0.03	-	-

The first layer Tree (Tree 1) is usually regarded as an unconditional dependency structure, which describes the most basic binary relationship between variables, independent of the conditions of other variables. The second level, Tree (Tree 2), begins to consider conditional dependencies, where the relationship between a pair of variables is affected by another variable. The third layer, Tree (Tree 3), goes further and examines more complex ternary conditional dependencies. The fourth layer, Tree (Tree 4), is the highest level of the D-Vine model and usually deals with conditional dependencies of four or higher order.

3.5 Measurement of risk spillover intensity

Formula (4) is used to calculate the conditional risk correlation matrix of the BRICS stock markets to measure the potential transmission effect of market changes in a single country on the market risks of other member countries. The results are shown in Table 2.

The table above shows the degree of interaction between the BRICS countries under certain conditions and measures the potential transmission effect to a single country market when multiple countries are at risk. For example, the first

Table 2 BRICS Stock market conditional risk correlation matrix ($\alpha = \beta = \theta = \mu = \gamma = 0.05$)

Affected country	Conditional risk	Mean(CoES)	min(CoES)	max(CoES)	var(CoES)
China	RussiaIndianSouth Africa Brazilian	3.83e-04	-8.37e-02	9.46e-02	1.39e-04
Russia	China IndianSouthAfrica Brazilian	4.30e-05	-1.72e-01	3.92e-01	4.19e-04
Indian	China RussiaSouth Africa Brazilian	-6.74e-04	-5.45e-02	6.04e-02	1.09e-04
SouthAfrica	China RussiaIndian Brazilian	-7.98e-05	-1.03e-01	4.56e-02	1.42e-04
Brazilian	China RussiaIndianSouth Africa	-1.54e-04	-7.93e-02	6.68e-02	1.95e-04

column of the first row represents the average loss that the Chinese stock market may face under the confidence level alpha =0.05 when the stock markets of Russia, India, South Africa, and Brazil fall into extreme risks. It can be seen from the table that when the affected country is Russia, the value is positive and large, indicating that when other countries fall into the extreme market, the Russian stock market tends to decline, and the volatility is large, indicating that the Russian economic market is unstable, which may be due to Russia's military action against Ukraine on February 24, 2022. In summary, although there are interactions among countries, the strength and direction of the impact vary. This provides a valuable perspective for understanding the complexity of economic interactions among BRICS countries and developing strategies to respond.

4 Conclusion

The purpose of this paper is to explore the relevant structures and risk spillovers among BRICS financial markets, with a particular focus on the interaction between China's financial markets and those of other BRICS countries. By using the non-parametric quantile method to fit the marginal distribution of returns and then using the D-Vine Copula model to depict the relevant structure, the paper then uses COES as a metric method to conduct an in-depth analysis of the risk spillover effect between China's financial market and other BRICS countries. According to the risk transmission path and effect strength, the average risk spillover effect of other BRICS countries on China and Russia is positive, while the average impact of other countries is negative. Russia has been most affected. Generally speaking, the mutual influence among BRICS countries is relatively stable, which may be related to the diversified cooperation among BRICS countries, such as the establishment of the BRICS Development Bank and the establishment of the monetary stability fund. However, while the BRICS countries have taken many measures to promote financial market stability, they also face challenges from uncertainties in the global economic environment.

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